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CEEX EW Corrections for $f\bar{f} \rightarrow f'\bar{f}'$ at LHC, Muon Colliders and FCC-ee as Realized in KK MC 4.22

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Abstract

With an eye toward the precision physics of the LHC, FCC-ee and possible high energy muon colliders, we present the extension of the CEEX (coherent exclusive exponentiation) realization of the YFS approach to resummation in our KK MC to include the processes $f\bar{f} \rightarrow f'\bar{f}'$, $f = \mu, \tau, q, \nu_\ell$, $f' = e, \mu, \tau, q, \nu_\ell$, $q = u, d, s, c, b, t$, $\ell = e, \mu, \tau$ with $f \neq f'$. After giving a brief summary of the CEEX theory with reference to the older EEX (exclusive exponentiation) theory, we illustrate theoretical results relevant to the LHC, FCC-ee, and possible muon collider physics programs.

Keywords: Coherent Exclusive Exponentiation EW Corrections Collider Physics

1. Introduction

In the context of the precision era for QCD in LHC physics (QCD precision tags $\leq 1\%$), higher order EW corrections are a necessity, as we have explained in Ref. [1]. Similarly, the muon collider physics program involves precision studies of the properties of the recently discovered [2, 3] BEH boson [4] and treatment of the effects of higher order EW corrections will be essential, as we illustrate in Ref. [1]. Building on our successful YFS/CEEX exponentiation [5, 6, 7, 8] realization in $\mathcal{K}\mathcal{K}\text{MC4.13}$ [9] in precision LEP, B-Factory and Tau-Charm factory physics, we have extended it to $\mathcal{K}\mathcal{K}\text{MC4.22}$ [1] wherein the incoming beams choice, previously restricted to e^+ , e^- , now allows $f\bar{f}$, $f = e, \mu, \tau, q, \nu_\ell$, $q = u, d, s, b$, $\ell = e, \mu, \tau$. We note that previous versions of $\mathcal{K}\mathcal{K}\text{MC}$ even though not adapted for the LHC were already found useful in estimations of theoretical systematic errors of other calculations [10, 11]. We also note the approaches of Refs. [12, 13, 14, 15, 16] to EW corrections to heavy gauge boson production at the LHC. In LEP studies [17] per mille level accuracy required higher order corrections beyond the exact $O(\alpha)$ EW correc-

tions. Our studies in ref. [1], briefly exhibited below, show that this is still the case so that the approaches in Ref. [12, 14, 16, 15] must be extended to higher orders for precision LHC studies. Observe that the QED parton shower approach used in Ref. [13] for these higher order corrections is intrinsically a $0 - p_T$ formalism and that ad hoc procedures, with varying degrees of success, are used to re-introduce non-zero p_T for the higher order effects whereas our CEEX exponentiation with exact $O(\alpha^2 L)$ corrections gives the higher effects systematically the non- p_T profile that is exactly correct in the soft limit to all orders in α . Here, we give a short summary in the next Section of the main features of YFS/CEEX exponentiation [7, 8] in the SM EW theory. In Sect. 3 we discuss the changes required to extend the incoming beam choices in the $\mathcal{K}\mathcal{K}\text{MC}$ to the more inclusive list of the SM fermions, present examples of theoretical results relevant for the LHC, FCC-ee [18] and possible muon collider [19] precision physics programs, and present our summary remarks.

2. Review of Standard Model calculations for e^+e^- annihilation with CEEX YFS exponentiation

We note that CEEX replaces the older EEX [6] – both are derived from the YFS theory [5]. Like what is also now featured in the MC's Herwig++ [20] and Sherpa [21] for particle decays, EEX, Exclusive EXponentiation, is very close to the original Yennie-Frautschi-Suura formulation. CEEX, Coherent EXclusive exponentiation, is actually an extension of the YFS theory. The coherence of CEEX is friendly to quantum coherence among the Feynman diagrams: we have the complete $|\sum_{diagr.}^n \mathcal{M}_i|^2$ rather than the often incomplete $\sum_{i,j}^n \mathcal{M}_i \mathcal{M}_j^*$. The proper treatment of narrow resonances, $\gamma \oplus Z$ exchanges, $t \oplus s$ channels, ISR \oplus FSR, angular ordering, etc. are all readily obtained as a consequence. Examples of the EEX formulation are KORALZ/YFS2, BHLUMI, BHWIDE, YFSWW, KoralW

and YFSZZ in our MC event generator approach; the only example of the CEEX formulation is $\mathcal{K}\mathcal{K}\text{MC}$.

For the process $e^-(p_1, \lambda_1) + e^+(p_2, \lambda_2) \rightarrow f(q_1, \lambda'_1) + \bar{f}(q_2, \lambda'_2) + \gamma(k_1, \sigma_1) + \dots + \gamma(k_n, \sigma_n)$ we illustrate CEEX schematically for the full scale CEEX $O(\alpha^r)$, $r=1,2$, master formula (realized in $\mathcal{K}\mathcal{K}\text{MC}$ for ISR and FSR up to $O(\alpha^2)$) which for the polarized total cross section reads

$$\sigma^{(r)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_n(p_a + p_b; p_c, p_d, k_1, \dots, k_n) \quad (1)$$

$$\times e^{2\alpha\mathcal{R}B_4} \sum_{\sigma_i, \lambda, \bar{\lambda}} \sum_{i,j,l,m=0}^3 \hat{\varepsilon}_a^i \hat{\varepsilon}_b^j \sigma_{\lambda_a \bar{\lambda}_a}^i \sigma_{\lambda_b \bar{\lambda}_b}^j \quad (2)$$

$$\times \mathfrak{M}_n^{(r)} \left(\frac{pk_1}{\lambda \sigma_1} \frac{k_2}{\sigma_2} \dots \frac{k_n}{\sigma_n} \right) \left[\mathfrak{M}_n^{(r)} \left(\frac{pk_1}{\bar{\lambda} \sigma_1} \frac{k_2}{\sigma_2} \dots \frac{k_n}{\sigma_n} \right) \right]^* \sigma_{\bar{\lambda}_c \lambda_c}^l \sigma_{\lambda_d \bar{\lambda}_d}^m \hat{h}_c^l \hat{h}_d^m. \quad (3)$$

The CEEX amplitudes are

$$\begin{aligned} \mathfrak{M}_n^{(1)} \left(\frac{pk_1}{\lambda \sigma_1} \dots \frac{k_n}{\sigma_n} \right) &= \sum_{\emptyset \in \mathcal{P}} \prod_{i=1}^n \mathfrak{s}_{[i]}^{\{\emptyset_i\}} \left\{ \hat{\beta}_0^{(1)}(p; X_{\emptyset}) + \sum_{j=1}^n \frac{\hat{\beta}_{1\{\emptyset_j\}}^{(1)}(p_{k_j}; X_{\emptyset})}{\mathfrak{s}_{[j]}^{\{\emptyset_j\}}} \right\} \\ \mathfrak{M}_n^{(2)} \left(\frac{pk_1}{\lambda \sigma_1} \dots \frac{k_n}{\sigma_n} \right) &= \sum_{\emptyset \in \mathcal{P}} \prod_{i=1}^n \mathfrak{s}_{[i]}^{\{\emptyset_i\}} \left\{ \hat{\beta}_0^{(2)}(p; X_{\emptyset}) + \sum_{j=1}^n \frac{\hat{\beta}_{1\{\emptyset_j\}}^{(2)}(p_{k_j}; X_{\emptyset})}{\mathfrak{s}_{[j]}^{\{\emptyset_j\}}} + \sum_{1 \leq j < l \leq n} \frac{\hat{\beta}_{2\{\emptyset_j, \emptyset_l\}}^{(2)}(p_{k_j, k_l}; X_{\emptyset})}{\mathfrak{s}_{[j]}^{\{\emptyset_j\}} \mathfrak{s}_{[l]}^{\{\emptyset_l\}}} \right\}. \end{aligned} \quad (4)$$

See refs. [7, 8] for the detailed IR function definitions and for many implications such as the $\mathcal{K}\mathcal{K}\text{MC}$ precision tag $\Delta\sigma/\sigma = 0.2\%$ at LEP established using the results from refs. [22, 23, 24]. Extension of $\mathcal{K}\mathcal{K}\text{MC}$ to the LHC, FCC and the muon collider awaits to their physics such per mille precision.

3. Including the processes $f\bar{f} \rightarrow f'\bar{f}'$, $f = \mu, q, \nu_\ell$, $f' = \ell, \nu_\ell, q, q = u, d, s, c, b$, $\ell = e, \mu, \tau$, $f \neq f'$, in $\mathcal{K}\mathcal{K}\text{MC}$

As we explain in ref. [1], we have extended the matrix elements, residuals, and IR functions in (1) to the case where we substitute the e^- , e^+ EW charges by the EW charges of the new beam particles f , \bar{f} and we substitute the mass m_e everywhere by m_f ¹. The resulting

new version of $\mathcal{K}\mathcal{K}\text{MC}$ is version 4.22. We have made considerable cross-checks [1] both during and after the extension, as we now illustrate.

In the most important cross-check, we exhibit in Tab. 1 here that, for the $e^+e^- \rightarrow \mu^+\mu^-$ process, $\mathcal{K}\mathcal{K}\text{MC}$ 4.22 reproduces the results in the corresponding $\sqrt{s} = 189\text{GeV}$ studies done in Ref. [7] for the dependence of the CEEX calculated cross section and A_{FB} on the energy cut-off on $v = 1 - s'/s$ where $s' = M_{\mu\bar{\mu}}^2$ is the invariant mass of the $\mu\bar{\mu}$ -system. This and its companion results given in ref. [1] show that our introduction of the new beams has not spoiled the precision of the $\mathcal{K}\mathcal{K}\text{MC}$ for the incoming e^+e^- state.

Proceeding pedagogically, especially given the interest in muon collider precision physics [19], we consider next the process $\mu^+\mu^- \rightarrow e^+e^-$ as our first new beam scenario, again at $\sqrt{s} = 189\text{GeV}$ to have as a reference the usual incoming e^+e^- annihilation scenario. In this new $\mu^+\mu^-$ scenario, while the EW

¹We advise the reader that especially in the QED radiation module KarLud for the ISR in $\mathcal{K}\mathcal{K}\text{MC}$, see Ref. [9], some of the expressions had Q_e and m_e effectively hard-wired into them and these had to all be found and substituted properly.

v_{\max}	$\mathcal{K}\mathcal{K}\text{sem Refer.}$	$\mathcal{O}(\alpha^3)_{\text{EEX3}}$	$\mathcal{O}(\alpha^2)_{\text{CEEX intOFF}}$	$\mathcal{O}(\alpha^2)_{\text{CEEX}}$
$\sigma(v_{\max})$ [pb]				
0.01	1.6712 ± 0.0000	1.6736 ± 0.0018	1.6738 ± 0.0018	1.7727 ± 0.0021
0.10	2.5198 ± 0.0000	2.5205 ± 0.0020	2.5210 ± 0.0020	2.6009 ± 0.0024
0.30	3.0616 ± 0.0000	3.0626 ± 0.0022	3.0634 ± 0.0022	3.1243 ± 0.0026
0.50	3.3747 ± 0.0000	3.3745 ± 0.0022	3.3761 ± 0.0022	3.4254 ± 0.0026
0.70	3.7223 ± 0.0000	3.7214 ± 0.0022	3.7249 ± 0.0022	3.7648 ± 0.0027
0.90	7.1430 ± 0.0000	7.1284 ± 0.0022	7.1530 ± 0.0022	7.1821 ± 0.0026
0.99	7.6136 ± 0.0000	7.5974 ± 0.0021	7.6278 ± 0.0021	7.6567 ± 0.0026
$A_{\text{FB}}(v_{\max})$				
0.01	0.5654 ± 0.0000	0.5661 ± 0.0012	0.5661 ± 0.0012	0.6121 ± 0.0014
0.10	0.5664 ± 0.0000	0.5667 ± 0.0009	0.5667 ± 0.0009	0.5931 ± 0.0011
0.30	0.5692 ± 0.0000	0.5694 ± 0.0008	0.5693 ± 0.0008	0.5864 ± 0.0010
0.50	0.5744 ± 0.0000	0.5744 ± 0.0008	0.5743 ± 0.0008	0.5870 ± 0.0009
0.70	0.5863 ± 0.0000	0.5858 ± 0.0007	0.5857 ± 0.0007	0.5953 ± 0.0008
0.90	0.3105 ± 0.0000	0.3107 ± 0.0004	0.3100 ± 0.0004	0.3176 ± 0.0004
0.99	0.2851 ± 0.0000	0.2856 ± 0.0003	0.2848 ± 0.0003	0.2918 ± 0.0004

Table 1: Energy cut-off study of total cross section σ and charge asymmetry A_{FB} for annihilation process $e^-e^+ \rightarrow \mu^-\mu^+$, at $\sqrt{s}=189\text{GeV}$. Energy cut: $v < v_{\max}$, $v = 1 - M_{f\bar{f}}^2/s$. Scattering angle for A_{FB} is θ^* (defined in Phys. Rev. **D41**, 1425 (1990)). No cut in θ^* . E-W corr. in $\mathcal{K}\mathcal{K}$ according to DIZET 6.x [12, 22]. In addition to CEEX matrix element, results are also shown for $\mathcal{O}(\alpha^3)_{\text{LL}}$ EEX3 matrix element without ISR \otimes FSR interf. $\mathcal{K}\mathcal{K}\text{sem}$ is a semi-analytical program, part of $\mathcal{K}\mathcal{K}\text{MC}$.

charges are all the same, the ISR probability to radiate factor $\gamma_e = \frac{2\alpha}{\pi} (\ln(s/m_e^2) - 1) \cong 0.114$ becomes $\gamma_\mu = \frac{2\alpha}{\pi} (\ln(s/m_\mu^2) - 1) \cong 0.0649$. This means that we expect the EW effects where the photonic corrections dominate to show reduction in size for ISR dominated regimes, the same size for the IFI dominated regimes. This is borne-out by the results in Tab. 6 and the companion results in ref. [1], which together provide a precision tag of 0.2% at an energy cut of 0.6. Precision results for EW effects would be available for the muon collider physics as it will be discussed elsewhere [25].

Turning next to the case of incoming quark anti-quark beams and proceeding as indicated above, for the process $u\bar{u} \rightarrow \mu^-\mu^+$ we obtain the results in Tab. 2 here and the companion results given in ref. [1], from which we get the precision tag .08% for the energy cut 0.6. This satisfies the requirements of precision LHC studies.

The third step in our extension is the introduction of PDF's for the quark beams. This is currently done in a hard-wired way using the beamsstrahlung module in the $\mathcal{K}\mathcal{K}\text{MC}$. A more algorithmic formulation of this part of the extension is in progress [25]. In Appendix A of ref. [1], sample output (three events in the LUND MC format) is given from a run of $\mathcal{K}\mathcal{K}\text{MC}$ version 4.22 for $pp \rightarrow u\bar{u} \rightarrow l^-l^+ + n\gamma$ where simple parton distribution functions (PDF's) of u and \bar{u} quarks in the proton are replacing beamsstrahlung distributions (see function BornV.RhoFoamC in the source code). The proton remnants are now represented by two photons in the event record with the exactly zero transverse mo-

v_{\max}	$\mathcal{K}\mathcal{K}\text{sem Refer.}$	$\mathcal{O}(\alpha^3)_{\text{EEX3}}$	$\mathcal{O}(\alpha^2)_{\text{CEEX intOFF}}$	$\mathcal{O}(\alpha^2)_{\text{CEEX}}$
$\sigma(v_{\max})$ [pb]				
0.01	1.2714 ± 0.0000	1.2718 ± 0.0009	1.2718 ± 0.0009	1.2191 ± 0.0009
0.10	1.6178 ± 0.0000	1.6175 ± 0.0010	1.6175 ± 0.0010	1.5792 ± 0.0010
0.30	1.8058 ± 0.0000	1.8053 ± 0.0010	1.8054 ± 0.0010	1.7784 ± 0.0010
0.50	1.9026 ± 0.0000	1.9018 ± 0.0010	1.9021 ± 0.0010	1.8815 ± 0.0011
0.70	2.0099 ± 0.0000	2.0084 ± 0.0010	2.0094 ± 0.0010	1.9938 ± 0.0011
0.90	3.3101 ± 0.0000	3.3023 ± 0.0010	3.3120 ± 0.0010	3.2993 ± 0.0010
0.99	3.3961 ± 0.0000	3.3881 ± 0.0010	3.3995 ± 0.0010	3.3872 ± 0.0010
$A_{\text{FB}}(v_{\max})$				
0.01	0.6788 ± 0.0000	0.6787 ± 0.0009	0.6787 ± 0.0009	0.6548 ± 0.0009
0.10	0.6791 ± 0.0000	0.6790 ± 0.0008	0.6790 ± 0.0008	0.6656 ± 0.0008
0.30	0.6799 ± 0.0000	0.6798 ± 0.0007	0.6798 ± 0.0007	0.6713 ± 0.0007
0.50	0.6809 ± 0.0000	0.6806 ± 0.0007	0.6806 ± 0.0007	0.6743 ± 0.0007
0.70	0.6800 ± 0.0000	0.6794 ± 0.0006	0.6793 ± 0.0006	0.6749 ± 0.0007
0.90	0.4417 ± 0.0000	0.4415 ± 0.0004	0.4407 ± 0.0004	0.4366 ± 0.0004
0.99	0.4285 ± 0.0000	0.4283 ± 0.0004	0.4274 ± 0.0004	0.4238 ± 0.0004

Table 2: Study of total cross section $\sigma(v_{\max})$ and charge asymmetry $A_{\text{FB}}(v_{\max})$, $u\bar{u} \rightarrow \mu^-\mu^+$, at $\sqrt{s} = 189\text{GeV}$. See Table 1 for definition of the energy cut v_{\max} , scattering angle and M.E. type,

mentum which were formerly beamsstrahlung photons (temporary fix). The multiple photon event in the sample output with a photon of $p_T = 31.6\text{ GeV}$ emphasizes the need to use $\mathcal{K}\mathcal{K}\text{MC}$ 4.22 to address the systematics of the current treatment of QED ISR at LHC with QED PDF's [25]. See ref. [1] for further discussion of this sample output.

Further development of $\mathcal{K}\mathcal{K}\text{MC}$, such as its inclusion in QCD parton shower MC's [26], is in progress as discussed in ref. [1]. Here we would also note application of $\mathcal{K}\mathcal{K}\text{MC}$ as it stands to new colliding beam devices such as FCC-ee [18], where we show in Fig. 1 the QED correction predicted by $\mathcal{K}\mathcal{K}\text{MC}$ for $\sigma(v\bar{v}\gamma)/\sigma(\mu\bar{\mu}\gamma)$ as a probe of the invisible Z width at 161 GeV with the cuts as prescribed in ref. [27]. The QED corrections seem to cancel to $\sim 0.03\%$: in the Z return regime, we get the initial theoretical precision error estimate $\Delta N_{\text{V}}^{th} \cong 0.00045$, consistent with the goals of the FCC-ee [27]. Further studies are needed to finalize this result [25]. To sum up, $\mathcal{K}\mathcal{K}\text{MC}$ is still alive and still useful. We thank Prof. I. Antoniadis for the support of the CERN TH Unit while this work was completed. This work is partly supported by the Polish National Science Centre grant DEC-2011/03/B/ST2/02632.

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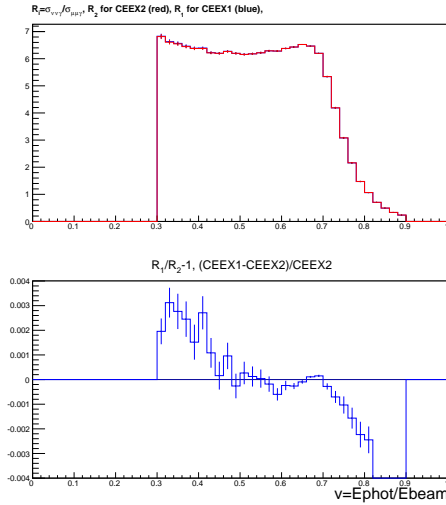


Figure 1: QED correction estimate in $\sigma(v\bar{\nu}\gamma)/\sigma(\mu\bar{\mu}\gamma)$: QED corrections seem to cancel dramatically in this ratio and drop to $\sim 0.03\%$. This preliminary result is undergoing further tests [25].

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